

# Angular analysis of $B \rightarrow J/\psi K_1$ : towards a model independent determination of the photon polarization with $B \rightarrow K_1 \gamma$

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## Abstract

We propose a model independent extraction of the hadronic information needed to determine the photon polarization of the  $b \rightarrow s\gamma$  process by the method utilizing the  $B \rightarrow K_1 \gamma \rightarrow K\pi\pi\gamma$  angular distribution. We show that exactly the same hadronic information can be obtained by using the  $B \rightarrow J/\psi K_1 \rightarrow J/\psi K\pi\pi$  channel, which leads to a much higher precision.

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## 1 Introduction

The circular polarization of the photon in the  $b \rightarrow s\gamma$  process has a unique sensitivity to new physics, namely to the right-handed charged current (see e.g. [1–3]). While it is a very fundamental observable, the experimental determination of the photon polarization was not achieved at a high precision in the previous  $B$  factory experiments. Therefore, this is a very important challenge for LHCb as well as for the upgrade of  $B$  factory, Belle II experiment. Various theoretical ideas to measure the photon polarization have been proposed (pioneered by [4–7] and followed by [8–11]) and many experimental efforts are currently on-going [12]. Since the photon polarization measurement determine the Wilson coefficient  $C_7^{(\prime)}$ , it will have an important consequence to the global fit as well [13].

Recently the LHCb collaboration has presented an interesting result [14] on the so-called up-down asymmetry of the  $B \rightarrow K\pi\pi\gamma$  decay, originally proposed in [6, 7]. The up-down asymmetry, which is the difference of the number of events with photon emitted above and below the  $K\pi\pi$  decay plane in the  $K\pi\pi$  reference frame, can indeed provide the information on the photon polarization. The basic idea is to determine the photon polarization by measuring the  $K_1$  polarization, which is correlated with the photon polarization, through its angular distribution in the  $B \rightarrow K\pi\pi\gamma$  decay.

To determine the photon polarization from the LHCb result, we need the detailed prediction of the  $K_1 \rightarrow K\pi\pi$  strong decay. In our previous works [8, 15], we have obtained this information by using the other experimental results, mainly the isobar model description from the ACCMOR collaboration [16], complemented by the theoretical model computation using the  $^3P_0$  model [17]. The  $B \rightarrow K_1(1270)\gamma \rightarrow K\pi\pi\gamma$  channel, different from the  $K_1(1400)$  channel, requires various unconventional treatments and unfortunately, our conclusion is that there are certain uncertainties remaining to describe this channel. The main difficulties are (see [15] for the detailed discussions) :

- the existence of two intermediate processes,  $K_1(1270) \rightarrow K^*\pi$  and  $K_1(1270) \rightarrow K\rho$ , with the latter being just on the edge of the  $K\rho$  phase space and having however a large branching ratio. Quasi-threshold effects must be taken into account.
- furthermore, as we found, the final estimation of photon polarization is also sensitive to the contribution of the  $K_1(1270)$  decay channels with scalar isobars,  $K_1(1270) \rightarrow K(\pi\pi)_{S\text{-wave}}$  or  $K_1(1270) \rightarrow (K\pi)_{S\text{-wave}}\pi$ , which are not well determined, neither by experiment nor by theory.

These problems must be solved in the future with more detailed analysis of  $K_1$  resonances, which are produced from  $B$ ,  $\tau$  or  $J/\psi$  decays.

In this article, we rather propose a model independent approach to circumvent the problem. In all the previous works, only a partial angular distribution was considered, i.e. taking into account only one  $\theta$  angle. We show in this article that with a more complete angular description, the information on the  $K_1$  decay needed for photon polarization determination can be extracted directly from  $B \rightarrow K\pi\pi + \gamma$  decay. That is, using the angles involving not only the  $\cos\theta$  like distribution which yields the up-down asymmetry, but also the azimuthal angle  $\phi$  dependence, we can obtain the full hadronic information without the isobar model description of the resonances.

In fact, with the limited statistics available for  $B \rightarrow K\pi\pi + \gamma$ , this method is currently difficult. On the other hand, it turns out that we can obtain the same hadronic information from another channel  $B \rightarrow K\pi\pi + J/\psi$  where two orders of magnitudes higher statistics, with respect to the photon channel, is available [18]. We show that the full angular distribution measurement allows us to separate the  $B$  decay and  $K_1$  decay parts so that we can extract the same hadronic information from the  $B \rightarrow K\pi\pi + J/\psi$  decay.

For the moment, for a simpler illustration of the approach, we consider the case of only one  $K_1$  resonance, which may be practically supported by the fact that  $B \rightarrow K_1(1270)\gamma$  seems largely dominant over  $B \rightarrow K_1(1400)\gamma$  [19].

The rest of the article is organized as follows: in section 2, we introduce the decay amplitudes of  $B \rightarrow K_1 J/\psi$  and  $B \rightarrow K_1 \gamma$  with  $K_1$  decaying to  $K\pi\pi$ . In section 3, we derive the angular distributions for these decays. Then, we demonstrate in section 4 that the hadronic information we need to determine the photon polarization in  $B \rightarrow K_1 \gamma$  can be obtained directly from the measurement of angular coefficients in  $B \rightarrow K_1 J/\psi$  and/or  $B \rightarrow K_1 \gamma$ , and we conclude in section 5.

## 2 The decay amplitudes and rates

The four body decay rate can be written as the product of the decay rates of  $B \rightarrow K_{1s_z} V_{s_z}$  and  $K_{1s_z} \rightarrow K\pi\pi$  summed over the different  $V$  polarizations <sup>\*</sup> :

$$\begin{aligned} d\Gamma_4^V(s) &\equiv d\Gamma(B \rightarrow K_1 V \rightarrow (K\pi\pi)V)_s \\ &= \sum_{s_z} \frac{(2\pi)^4}{2M_B} \left| \mathcal{M}_{s_z}^V(B \rightarrow K_{1s_z} V \rightarrow (K\pi\pi)V)_s \right|^2 (2\pi)^3 ds d\Phi_2 d\Phi_3, \end{aligned} \quad (1)$$

where  $s_z$  is the polarization of  $V = J/\psi, \gamma$  :

$$s_z = 0, \pm 1 \quad (\text{for } V = J/\psi), \quad s_z = \pm 1 \quad (\text{for } V = \gamma). \quad (2)$$

Here,  $B$  can be  $B^\pm$ ,  $B^0$  or  $\bar{B}^0$ . Denoting the amplitude of  $B \rightarrow K_1(s)V$  as  $\mathcal{A}_{s_z}(s)$  and of  $K_1(s) \rightarrow K\pi\pi$  as  $\epsilon_{K_{1s_z}}^\mu \mathcal{J}_\mu$ , one can write :

$$\mathcal{M}_{s_z}^V(B \rightarrow K_{1s_z} V \rightarrow (K\pi\pi)V)_s = \frac{\mathcal{A}_{s_z}^V(s) \times (\epsilon_{K_{1s_z}}^\mu \mathcal{J}_\mu(s_{13}, s_{23})_s)}{(s - m_{K_1}^2) + im_{K_1} \Gamma_{K_1}(s)}. \quad (3)$$

In the following, we consider only  $K_1 = K_1(1270)$  for simplicity, though it can be readily extended to include  $K_1(1400)$ . The propagator of the  $K_1$ , which is parametrized here as Breit-Wigner function, is introduced in order to use the  $K\pi\pi$  invariant mass  $m_{K\pi\pi} \equiv \sqrt{s}$  as the varying  $K_1$  mass. The  $K_1$  rest frame is meant as the actual  $K\pi\pi$  system. This is not a convention, but an assumption on the off-shell extrapolation of amplitudes, partially justified by unitarity. Note that this implies that the Dalitz plot  $(s_{13}, s_{23})$  depends on  $s$  as well.

In Eq. (3), the full kinematical variable dependence of  $\mathcal{J}$  is left implicit but it can be displayed with help of two form factors as  $\mathcal{C}_{1,2}$  [8]:

$$\mathcal{J}_\mu(s_{13}, s_{23})_s \equiv \mathcal{C}_1(s, s_{13}, s_{23}) p_{1\mu} - \mathcal{C}_2(s, s_{13}, s_{23}) p_{2\mu}. \quad (4)$$

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<sup>\*</sup>We follow the PDG convention, i.e.  $\int_\Omega d\Phi_2 = \frac{1}{(2\pi)^5} \frac{|\vec{p}_V^*|}{2M_B}$ ,  $\int_\psi d\Phi_3 = \frac{1}{32(2\pi)^8} \frac{1}{s} ds_{13} ds_{23} d\phi d(\cos\theta)$ .

These form factors could be made explicit in a quasi-two-body approach to the  $K_1$  decay [?, ?]. Here, on the contrary, we want to determine them in a model independent way by using the experimental data to avoid the ambiguities described in the introduction.

### 3 Angular distribution

Now, we define the probability density function (PDF) for a given value of  $s$ . First, the different transverse ( $s_z = \pm$ ) and the longitudinal ( $s_z = 0$ ) polarizations of  $V$  state do not interfere, thus the decay rate is written as<sup>†</sup> :

$$\begin{aligned} \frac{d\Gamma(B \rightarrow K_1 V \rightarrow (K\pi\pi)V)_s}{ds_{13}ds_{23}d(\cos\theta)d\phi} &= \frac{(2\pi)^4}{2M_B} (2\pi)^3 ds \frac{1}{(2\pi)^5} \frac{|\vec{p}_V^*|}{2M_B} \\ &\times \frac{1}{32(2\pi)^8 s} \left| \frac{1}{(s - m_{K_1}^2) + im_{K_1}\Gamma_{K_1}(s)} \right|^2 \\ &\times \sum_{s_z} |\mathcal{A}_{s_z}^V(s)|^2 \left| \vec{\epsilon}_{K_1 s_z} \cdot \vec{\mathcal{J}}_{K_1}(s_{13}, s_{23})_s \right|^2, \end{aligned} \quad (5)$$

where  $\vec{p}_V^*$  is the three momentum of  $V$  in the  $B$  reference frame, while the  $K_1$  polarization vector  $\vec{\epsilon}_{K_1}$  and  $\vec{\mathcal{J}}_{K_1}$  are defined in the  $K_1$  reference frame. Note that in Eq. (5), the *width* in the denominator could also be related to  $\vec{\mathcal{J}}_{K_1}$ , except, we have to add all charge combinations,  $K_1^+ \rightarrow K^+\pi^+\pi^-$  and  $K_1^+ \rightarrow K^0\pi^+\pi^0$  for  $K_1^+$  and  $K_1^0 \rightarrow K^+\pi^0\pi^-$  and  $K_1^0 \rightarrow K^0\pi^+\pi^-$  for  $K_1^0$  (and similar for the charge conjugations).

The PDF  $\mathcal{W}^V(s_{13}, s_{23}, \cos\theta, \phi)_s$  is obtained from Eq. (5) and is normalized as :

$$\int ds_{13} \int ds_{23} \int d(\cos\theta) \int d\phi \mathcal{W}^V(s_{13}, s_{23}, \cos\theta, \phi)_s = 1. \quad (6)$$

Thus, the PDF can be written in terms of the squared decay amplitudes, which are the functions of the kinematical variables we are interested in, without the irrelevant pre-factors :

$$\mathcal{W}^V(s_{13}, s_{23}, \cos\theta, \phi)_s = \frac{\sum_{s_z} |\mathcal{A}_{s_z}^V(s)|^2 \left| \vec{\epsilon}_{K_1 s_z} \cdot \vec{\mathcal{J}}_{K_1}(s_{13}, s_{23})_s \right|^2}{\int ds_{13} \int ds_{23} \int d(\cos\theta) \int d\phi \sum_{s_z} |\mathcal{A}_{s_z}^V(s)|^2 \left| \vec{\epsilon}_{K_1 s_z} \cdot \vec{\mathcal{J}}_{K_1}(s_{13}, s_{23})_s \right|^2} \quad (7)$$

Next we make explicit the angular distribution of  $\mathcal{W}^V$  (the definition of the coordinate system and angles is given in the Appendix) :

$$\mathcal{W}^V(s_{13}, s_{23}, \cos\theta, \phi)_s \equiv a^V + (a_1^V + a_2^V \cos 2\phi + a_3^V \sin 2\phi) \sin^2 \theta + b^V \cos \theta, \quad (8)$$

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<sup>†</sup>For  $V = J/\psi$ , we integrate over the  $J/\psi$  decay angle here so that the interference term disappears.

where the angular coefficients depend on the Dalitz variables and fixed value of  $s$ . They can be written as :

$$a^V(s, s_{13}, s_{23}) = N_s^V \xi_a^V [|c_1|^2 + |c_2|^2 - 2\text{Re}(c_1 c_2^*) \cos \delta] , \quad (9)$$

$$a_1^V(s, s_{13}, s_{23}) = N_s^V \xi_{a_i}^V [|c_1|^2 + |c_2|^2 - 2\text{Re}(c_1 c_2^*) \cos \delta] , \quad (10)$$

$$a_2^V(s, s_{13}, s_{23}) = N_s^V \xi_{a_i}^V [(|c_1|^2 + |c_2|^2) \cos \delta - 2\text{Re}(c_1 c_2^*)] , \quad (11)$$

$$a_3^V(s, s_{13}, s_{23}) = N_s^V \xi_{a_i}^V [(|c_1|^2 - |c_2|^2) \sin \delta] , \quad (12)$$

$$b^V(s, s_{13}, s_{23}) = -N_s^V \xi_b^V [2\text{Im}(c_1 c_2^*) \sin \delta] , \quad (13)$$

where the factor  $N_s^V > 0$  is the normalization factor, which is equal to the inverse of the denominator of Eq. (7).

The  $\xi$ 's represent the  $B \rightarrow K_1 V$  decay, and thus, depend only on  $s$

$$\begin{aligned} \xi_a^V(s) &\equiv \frac{|\mathcal{A}_+^V(s)|^2 + |\mathcal{A}_-^V(s)|^2}{2} , \\ \xi_{a_i}^V(s) &\equiv \frac{-(|\mathcal{A}_+^V(s)|^2 + |\mathcal{A}_-^V(s)|^2) + 2|\mathcal{A}_0^V(s)|^2}{4} , \\ \xi_b^V(s) &\equiv \frac{|\mathcal{A}_+^V(s)|^2 - |\mathcal{A}_-^V(s)|^2}{2} . \end{aligned} \quad (14)$$

In fact, for  $V = \gamma$ , the longitudinal amplitude vanishes ( $\mathcal{A}_0^\gamma = 0$ ), which simplifies the above expressions, giving as a result  $a^\gamma = -2a_1^\gamma$ .

The coefficients  $c_{1,2}$  are related to the form factors in Eq. (4) as :

$$c_1(s, s_{13}, s_{23}) = \mathcal{C}_1(s, s_{13}, s_{23}) |\vec{p}_1| , \quad c_2(s, s_{13}, s_{23}) = \mathcal{C}_2(s, s_{13}, s_{23}) |\vec{p}_2| ,$$

where we wrote explicitly the Dalitz variables dependence. The angle  $\delta$  (with  $0 < \delta < \pi$ ) is defined as

$$\cos \delta = \frac{\vec{p}_1 \cdot \vec{p}_2}{|\vec{p}_1| |\vec{p}_2|} .$$

Let us also remind that all the relevant kinematical variables can be expressed in terms of the Dalitz variables :

$$|\vec{p}_{1,2}|^2 = E_{1,2}^2 - m_{1,2}^2 , \quad \vec{p}_1 \cdot \vec{p}_2 = E_1 E_2 - \frac{s_{12} - m_1^2 - m_2^2}{2} , \quad E_{1,2} = \frac{s - s_{23,13} + m_{1,2}^2}{2\sqrt{s}} .$$

## 4 Photon polarization : relating the $B \rightarrow K_1 \gamma$ and $B \rightarrow K_1 J/\psi$ amplitudes

The photon polarization in the  $B \rightarrow K_1 \gamma$  process which we want to determine is defined as following :

$$\lambda_\gamma \equiv \frac{|\mathcal{A}_+^\gamma(s)|^2 - |\mathcal{A}_-^\gamma(s)|^2}{|\mathcal{A}_+^\gamma(s)|^2 + |\mathcal{A}_-^\gamma(s)|^2} , \quad (15)$$

where in the SM,  $\lambda_\gamma \simeq +1(-1)$  for  $B^0, B^+(\bar{B}^0, B^-)$ . In this article, we do not discuss the so-called charm loop contributions, which may differentiate slightly  $\lambda_\gamma$  from  $\pm 1$ . Under this assumption, the  $s$ -dependence of  $\mathcal{A}_\pm^\gamma(s) \propto T_1(s)$ , where  $T_1$  is the  $B \rightarrow K_1$  hadronic form factor, is cancelled out in the ratios. Hence, one can write  $\xi_{a,a_i,b}^\gamma(s) = \xi_{a,a_i,b}^\gamma$ <sup>‡</sup>. Using Eq. (14), one can find

$$\lambda_\gamma = \frac{\xi_b^\gamma}{\xi_a^\gamma}. \quad (16)$$

In the following, we show that the  $\xi_{a,a_i,b}^\gamma$ 's can be indeed obtained from the measurement of  $a^V, a_i^V, a^\gamma, b^\gamma$  in a model independent way.

First, we obtain  $\xi_a^\gamma$  via :

$$\xi_a^\gamma = \frac{a^\gamma(s, s_{13}, s_{23})}{N_s^\gamma [|c_1|^2 + |c_2|^2 - 2\text{Re}(c_1 c_2^*) \cos \delta]}. \quad (17)$$

The term in the square brackets in the denominator is common for  $V = J/\psi, \gamma$  and can be obtained for given point of  $(s, s_{13}, s_{23})$  as

$$|c_1|^2 + |c_2|^2 - 2\text{Re}(c_1 c_2^*) \cos \delta = \frac{a^V(s, s_{13}, s_{23})}{N_s^V \xi_a^V(s)} = \frac{a_1^V(s, s_{13}, s_{23})}{N_s^V \xi_{a_i}^V(s)}. \quad (18)$$

Next, we determine  $\xi_b^\gamma$  from the experimental measurement of  $b^\gamma(s, s_{13}, s_{23})$  :

$$\xi_b^\gamma = -\frac{b^\gamma(s, s_{13}, s_{23})}{N_s^\gamma [2 \text{Im}(c_1 c_2^*) \sin \delta]}. \quad (19)$$

Now we obtain the denominator factor  $2\text{Im}(c_1 c_2^*) \sin \delta$ . By writing

$$\text{Im}(c_1 c_2^*) = \pm \sqrt{|c_1|^2 |c_2|^2 - [\text{Re}(c_1 c_2^*)]^2},$$

we find that we need to obtain independently these two factors,  $|c_1|^2 |c_2|^2$  and  $\text{Re}(c_1 c_2^*)$ , from the above equations. Then, by using Eqs. (10)-(12), we find

$$2 \text{Im}(c_1 c_2^*) \sin \delta = \pm \frac{1}{N_s^V \xi_{a_i}^V(s)} \sqrt{(a_1^V(s, s_{13}, s_{23}))^2 - (a_2^V(s, s_{13}, s_{23}))^2 - (a_3^V(s, s_{13}, s_{23}))^2} \quad (20)$$

Finally, the sign ambiguity remains, which can not be resolved at this point.

Now by inserting Eqs. (17)-(20) into Eq. (16), we can obtain the polarization parameter which we want to determine :

$$\lambda_\gamma = \frac{\xi_b^\gamma}{\xi_a^\gamma} = \mp \frac{b^\gamma(s, s_{13}, s_{23})}{a^\gamma(s, s_{13}, s_{23})} \times \frac{1}{\sqrt{1 - \left(\frac{a_2^V(s, s_{13}, s_{23})}{a_1^V(s, s_{13}, s_{23})}\right)^2 - \left(\frac{a_3^V(s, s_{13}, s_{23})}{a_1^V(s, s_{13}, s_{23})}\right)^2}}. \quad (21)$$

The right hand side of Eq. (21) is the main result of this paper. This equation implies :

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<sup>‡</sup>For the same reason, strictly speaking,  $\lambda_\gamma$  here is slightly different from the usual definition of  $\lambda_\gamma \equiv \frac{|C_+|^2 - |C_-|^2}{|C_+|^2 + |C_-|^2}$  where  $C_\pm$  represents only the short-distance  $b \rightarrow s\gamma$  decay.

- The photon polarization in  $B \rightarrow K_1\gamma$  can be obtained from the measurement of the angular coefficients  $a^\gamma(s, s_{13}, s_{23})$ ,  $b^\gamma(s, s_{13}, s_{23})$  which can be measured only via the standard  $\cos\theta$  distribution, together with the coefficients  $a_{1,2,3}^V(s, s_{13}, s_{23})$  which requires the azimuthal angle  $\phi$  distribution. The advantage is that the latter coefficients can be measured equally by using either  $B \rightarrow J/\psi K_1$  or  $B \rightarrow K_1\gamma$  decays. Therefore, we can take advantage of the much higher statistics of the  $J/\psi$  process.
- The final results depend only on the ratio of the angular coefficients so that there is no need for the normalization.
- The photon polarization  $\lambda_\gamma$  does not depend on  $s$  nor any Dalitz variables (except for the neglected charm contribution mentioned in the section 2), which implies that the expression in Eq. (21) is constant at any point of the  $(s, s_{13}, s_{23})$  plane. When we use the  $J/\psi$  to determine the denominator of this term, we simply need to map point by point on the Dalitz plane.
- Concerning the sign ambiguity, in practice, we may measure the absolute value of the polarization parameter  $|\lambda_\gamma|$ . In this way, we are left with the sign ambiguity of overall sign of  $\lambda_\gamma$  but we can neglect the sign variation of  $b^\gamma/a^\gamma$  term since  $\lambda_\gamma$  must be constant in the  $(s, s_{13}, s_{23})$  plane.

The third point has important consequence: arbitrary binning may lead to a variation of  $\lambda_\gamma$  depending on the Dalitz points. Having the large sample available in  $B \rightarrow K_1 J/\psi$  ( $\sim \mathcal{O}(10^3)$  events in the  $K_1(1270)$  region even at Belle [18], which means orders of magnitudes higher at LHCb), a high sensitivity to  $\lambda_\gamma$  is expected. Nevertheless, the reliability of method has to be confirmed with a Monte Carlo simulation. In particular, the optimization of the binning could be used by modeling the resonances in a crude manner.

## 5 Conclusions

The angular distribution in the polar angle  $\theta$  of the  $B \rightarrow K_{\text{res}}\gamma \rightarrow K\pi\pi\gamma$  process has recently been measured by the LHCb collaboration [14]. Among various kaonic resonances  $K_{\text{res}}$ , a large  $B \rightarrow K_1(1270)\gamma$  contribution has been identified, confirming the previous result [19]. The extraction of the  $b \rightarrow s\gamma$  photon polarization from this data requires a detailed knowledge of the  $K_1$  decays, in particular, the imaginary part of the product of the two form factors,  $\text{Im}(c_1 c_2^*)$ . The imaginary part is, in general, very sensitive to the resonance structure of the decay while there are many uncertainties in the resonance decay structure of  $K_1(1270)$ , especially due to i) the limited phase space for the main decay channel  $K_1(1270) \rightarrow \rho K$  resulting in strong distortion effects, ii) a possible  $K_1(1270) \rightarrow \kappa\pi$  contributions, neither well determined experimentally nor theoretically tractable.

In order to circumvent this problem, we propose a resonance model independent determination of the strong interaction factor  $\text{Im}(c_1 c_2^*)$ . This method requires the Dalitz plot of the angular coefficients including both polar and azimuthal angles. In this article, we have shown that the same Dalitz plot analysis can be also obtained through the  $B \rightarrow J/\psi K_1 \rightarrow J/\psi K \pi \pi$  channel. The  $B$  decay part of these two channels are very different while we found that we have enough observables to separate the  $B$  decay part. The realization of our proposal would require a detailed Monte Carlo studies, in particular by evaluating the binning effect.

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## A Kinematics of $B^+ \rightarrow V K_1^+ \rightarrow V K^+ \pi^+ \pi^-$ decay ( $V = J/\psi, \gamma$ )

In this section, we describe all the definitions of the kinematical variables. We use  $B^+ \rightarrow V K_1^+ \rightarrow V K^+ \pi^+ \pi^-$  decay as an example but one can obtain the similar formulae for other charge combinations. Throughout this article, we work in the  $K_1$  rest frame. We can move to the conventional  $B$  rest frame or any other frame simply by a Lorentz transformation. First, we assign the three momenta as

$$\pi^+(\vec{p}_1), \quad \pi^-(\vec{p}_2), \quad K^+(\vec{p}_3). \quad (22)$$

Now, we define a standard orthogonal frame, with respect to the spin direction of  $K_1$ , or  $V = J/\psi, \gamma$ . First, the  $Oz$  is defined as the  $V$  direction

$$\vec{e}_z = \frac{\vec{p}_V}{|\vec{p}_V|} = \frac{-\vec{p}_B}{|\vec{p}_B|}. \quad (23)$$

We define the axis perpendicular to the  $K\pi\pi$  decay plane by  $\vec{n}$ :

$$\vec{n} = \frac{\vec{p}_1 \times \vec{p}_2}{|\vec{p}_1 \times \vec{p}_2|}. \quad (24)$$

Then, the  $Oy$  is chosen as normal to the  $Oz$  and  $V = J/\psi, \gamma$  direction by

$$\vec{e}_y = \frac{\vec{p}_V \times \vec{n}}{|\vec{p}_V \times \vec{n}|}. \quad (25)$$



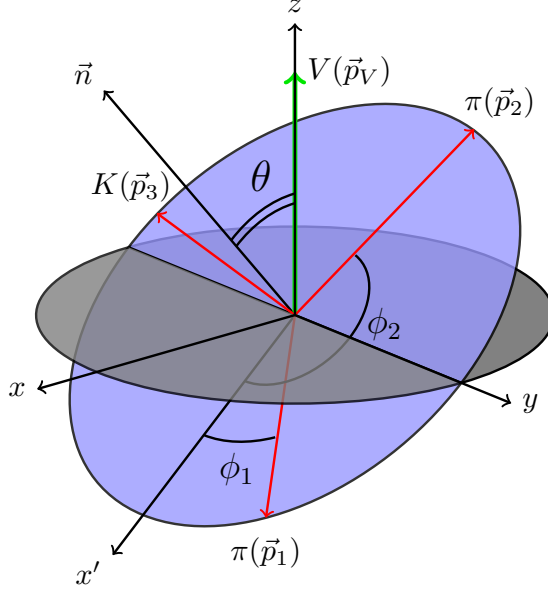


Figure 1: Kinematics of the  $B \rightarrow K_1(\rightarrow K\pi\pi)V$  decay.

Finally,  $Ox$  is then chosen as the normal to  $Oy$  and  $Oz$  :  $\vec{e}_x = \vec{e}_y \times \vec{e}_z$  .

One also defines a polar angle  $\theta$ , of  $\vec{n}$  with respect to the  $\vec{e}_z$  :

$$\cos \theta = \vec{e}_z \cdot \vec{n} \quad (26)$$

Let us here set a condition for  $\theta$  as

$$\vec{e}_x \cdot \vec{n} = \sin \theta > 0, \quad 0 < \theta < \pi. \quad (27)$$

Now we rotate  $\vec{e}_x$  onto the  $K\pi\pi$  decay plane and define the result as  $\vec{e}'_x$  which can be written as

$$\vec{e}'_x = \vec{e}_y \times \vec{n} \quad (28)$$

We can then define a second orthogonal frame, which is based on the  $K_1$  decay plane,  $\vec{e}', \vec{e}_y, \vec{n}$ . Defining  $\phi_{1,2}$  to be the azimuthal angle from the  $\vec{e}'_x$  axis in this  $(x', y)$  decay plane, the components of the pions three momenta,

$$\vec{p}_{1,2} = |\vec{p}_{1,2}|(\cos \phi_{1,2} \vec{e}'_x + \sin \phi_{1,2} \vec{e}_y), \quad (29)$$

can be expressed in terms of  $\theta, \phi_{1,2}$  in the standard frame as :

$$\begin{aligned} (\vec{p}_{1,2})_x &= |\vec{p}_{1,2}| \cos \theta \cos \phi_{1,2}, \\ (\vec{p}_{1,2})_y &= |\vec{p}_{1,2}| \sin \phi_{1,2}, \\ (\vec{p}_{1,2})_z &= -|\vec{p}_{1,2}| \sin \theta \cos \phi_{1,2}. \end{aligned} \quad (30)$$

The advantage is that the angles  $\theta, \phi_{1,2}$  are connected directly with the decay plane. We note that the linear combination of the  $\phi_{1,2}$  angles,

$$\delta \equiv \phi_2 - \phi_1, \quad (31)$$

is a function the Dalitz variables defined by

$$\begin{aligned} s &= (p_{K_1})^2 \\ s_{13} &= (p_1 + p_3)^2 = (p_{K_1} - p_2)^2, \\ s_{23} &= (p_2 + p_3)^2 = (p_{K_1} - p_1)^2, \\ s_{12} &= (p_1 + p_2)^2 = (p_{K_1} - p_3)^2. \end{aligned} \quad (32)$$

In the  $K_1$  rest frame,  $\vec{p}_{K_1} = 0$  and  $|\vec{p}_{1,2,3}|$  can be expressed in terms of  $s_{23}, s_{13}, s_{12}$  respectively. Since only two of them are independent, we choose  $s_{23}, s_{13}$  for symmetry. Then the relative angle between the three momenta of the two pions

$$\cos \delta = \frac{\vec{p}_1 \cdot \vec{p}_2}{|\vec{p}_1||\vec{p}_2|} = \frac{|\vec{p}_3|^2 - |\vec{p}_1|^2 - |\vec{p}_2|^2}{2|\vec{p}_1||\vec{p}_2|}, \quad (33)$$

is expressible in terms of  $s, s_{13}, s_{23}$ . The same holds for the other relative angles between the three momenta<sup>§</sup>. This means that the  $K\pi\pi$  system is rigid once the masses of the two  $K\pi$  subsystems have been chosen. It is still allowed to rotate however : if the normal is fixed by a definite  $\theta$ , there remains a free rotation of the rigid  $K\pi\pi$  system around  $\vec{n}$  in the decay plane. We choose the angle defining this rotation as :

$$\phi \equiv \frac{\phi_1 + \phi_2}{2}. \quad (34)$$

In this way, the angle  $\phi$  in the reference [6] is now fixed, which allows to perform definite calculations. Note that our definition is just one possible among many others while we have found it convenient because it simplifies the calculations.

Then, re-expressing  $\phi_{1,2}$  as

$$\phi_{1,2} = \phi \mp \frac{\delta}{2},$$

one can get the components of  $\vec{p}_{1,2}$  in Eq. (30), expressed in terms of  $\phi$  and the Dalitz variables.

## References

- [1] D. Becirevic, E. Kou, A. Le Yaouanc and A. Tayduganov, JHEP **1208** (2012) 090 doi:10.1007/JHEP08(2012)090 [arXiv:1206.1502 [hep-ph]].

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<sup>§</sup>We have furthermore  $0 < \delta < \pi, (\sin \delta > 0)$ , because the angles  $\phi_1, \phi_2$  are measured in the plane oriented by the normal  $\vec{n} = \vec{p}_1 \times \vec{p}_2 / |\vec{p}_1 \times \vec{p}_2|$ .

- [2] E. Kou, C. D. Lu and F. S. Yu, JHEP **1312** (2013) 102 doi:10.1007/JHEP12(2013)102 [arXiv:1305.3173 [hep-ph]].
- [3] N. Haba, H. Ishida, T. Nakaya, Y. Shimizu and R. Takahashi, JHEP **1503** (2015) 160 doi:10.1007/JHEP03(2015)160 [arXiv:1501.00668 [hep-ph]].
- [4] D. Atwood, M. Gronau and A. Soni, Phys. Rev. Lett. **79** (1997) 185 doi:10.1103/PhysRevLett.79.185 [hep-ph/9704272].
- [5] D. Atwood, T. Gershon, M. Hazumi and A. Soni, hep-ph/0701021.
- [6] M. Gronau, Y. Grossman, D. Pirjol and A. Ryd, Phys. Rev. Lett. **88** (2002) 051802 doi:10.1103/PhysRevLett.88.051802 [hep-ph/0107254].
- [7] M. Gronau and D. Pirjol, Phys. Rev. D **66** (2002) 054008 doi:10.1103/PhysRevD.66.054008 [hep-ph/0205065].
- [8] E. Kou, A. Le Yaouanc and A. Tayduganov, Phys. Rev. D **83** (2011) 094007 doi:10.1103/PhysRevD.83.094007 [arXiv:1011.6593 [hep-ph]].
- [9] F. Bishara and D. J. Robinson, JHEP **1509** (2015) 013 doi:10.1007/JHEP09(2015)013 [arXiv:1505.00376 [hep-ph]].
- [10] F. Muheim, Y. Xie and R. Zwicky, Phys. Lett. B **664** (2008) 174 doi:10.1016/j.physletb.2008.05.032 [arXiv:0802.0876 [hep-ph]].
- [11] L. Oliver, J.-C. Raynal and R. Sinha, Phys. Rev. D **82** (2010) 117502 doi:10.1103/PhysRevD.82.117502 [arXiv:1007.3632 [hep-ph]].
- [12] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D **78** (2008) 071102 doi:10.1103/PhysRevD.78.071102 [arXiv:0807.3103 [hep-ex]]. Y. Ushiroda *et al.* [Belle Collaboration], Phys. Rev. D **74** (2006) 111104 doi:10.1103/PhysRevD.74.111104 [hep-ex/0608017]. J. Li *et al.* [Belle Collaboration], Phys. Rev. Lett. **101** (2008) 251601 doi:10.1103/PhysRevLett.101.251601 [arXiv:0806.1980 [hep-ex]]. P. del Amo Sanchez *et al.* [BaBar Collaboration], arXiv:1512.03579 [hep-ex]. R. Aaij *et al.* [LHCb Collaboration], JHEP **1504** (2015) 064 doi:10.1007/JHEP04(2015)064 [arXiv:1501.03038 [hep-ex]].
- [13] S. Descotes-Genon, D. Ghosh, J. Matias and M. Ramon, JHEP **1106** (2011) 099 doi:10.1007/JHEP06(2011)099 [arXiv:1104.3342 [hep-ph]]. D. Becirevic and A. Tayduganov, Nucl. Phys. B **868** (2013) 368 doi:10.1016/j.nuclphysb.2012.11.016 [arXiv:1207.4004 [hep-ph]]. D. Becirevic and E. Schneider, Nucl. Phys. B **854** (2012) 321 doi:10.1016/j.nuclphysb.2011.09.004 [arXiv:1106.3283 [hep-ph]]. S. Jger

- and J. Martin Camalich, Phys. Rev. D **93** (2016) no.1, 014028  
doi:10.1103/PhysRevD.93.014028 [arXiv:1412.3183 [hep-ph]].
- [14] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **112** (2014) no.16, 161801  
doi:10.1103/PhysRevLett.112.161801 [arXiv:1402.6852 [hep-ex]].
- [15] A. Tayduganov, E. Kou and A. Le Yaouanc, Phys. Rev. D **85** (2012) 074011  
doi:10.1103/PhysRevD.85.074011 [arXiv:1111.6307 [hep-ph]].
- [16] C. Daum *et al.* [ACCMOR Collaboration], Nucl. Phys. B **187** (1981) 1.  
doi:10.1016/0550-3213(81)90114-0
- [17] A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Rev. D **8** (1973) 2223.  
doi:10.1103/PhysRevD.8.2223
- [18] H. Guler *et al.* [Belle Collaboration], Phys. Rev. D **83** (2011) 032005  
doi:10.1103/PhysRevD.83.032005 [arXiv:1009.5256 [hep-ex]].
- [19] H. Yang *et al.* [Belle Collaboration], Phys. Rev. Lett. **94** (2005) 111802  
doi:10.1103/PhysRevLett.94.111802 [hep-ex/0412039].